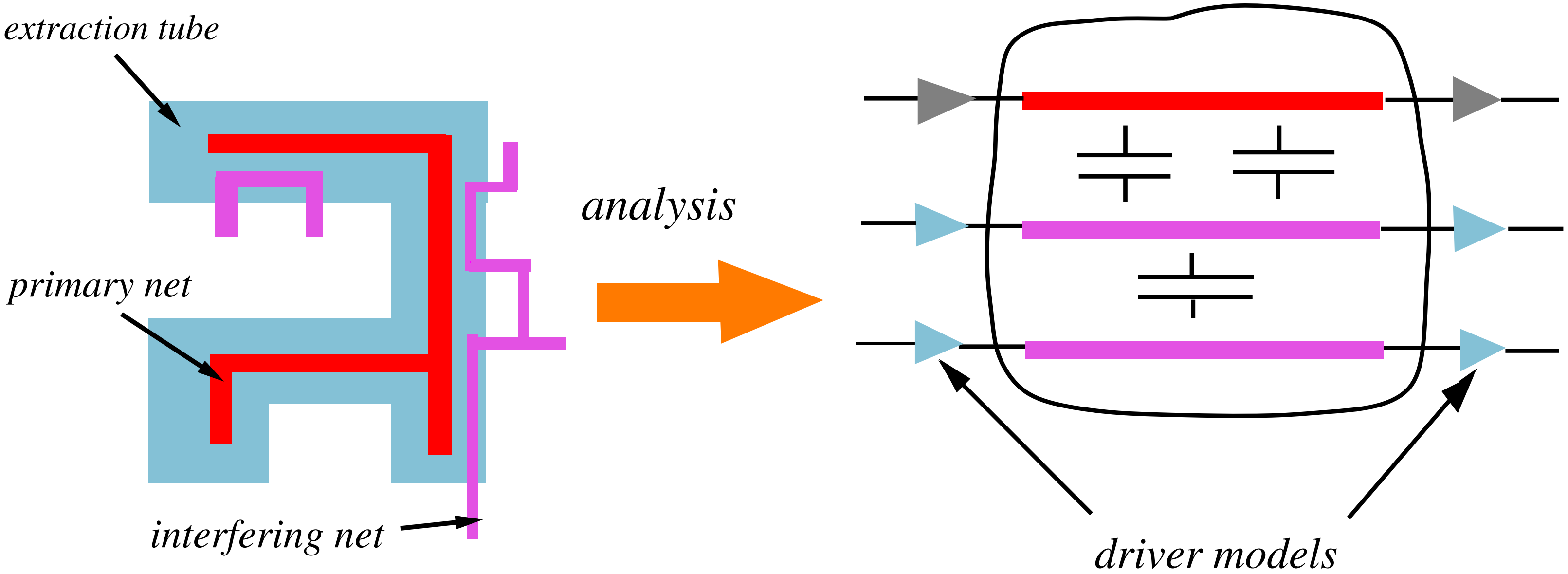
Reduction techniques

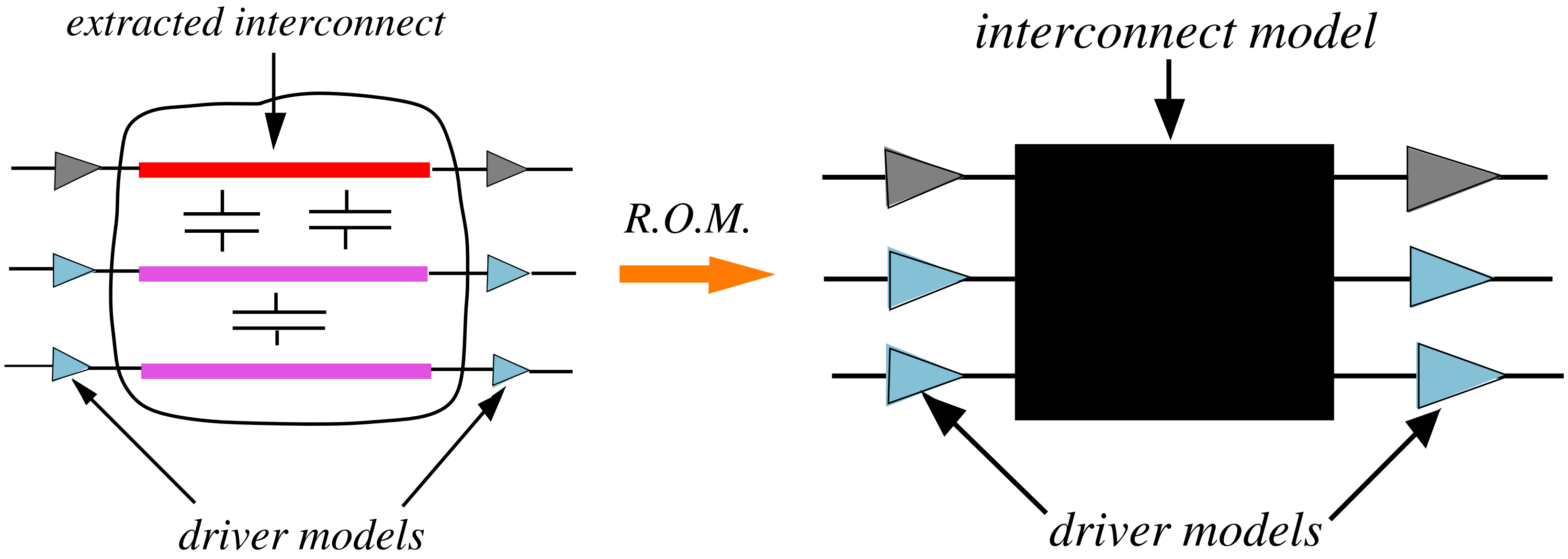
- Signal-integrity verification
- Formulation
- Frequency-domain transfer function
- Matrix-Padé approximation
- Krylov-subspace techniques
- Reduced-order models
- Properties of reduced-order models
- Trade-off of reduction techniques on the market
- Delay and cross-talk

Signal-integrity verification



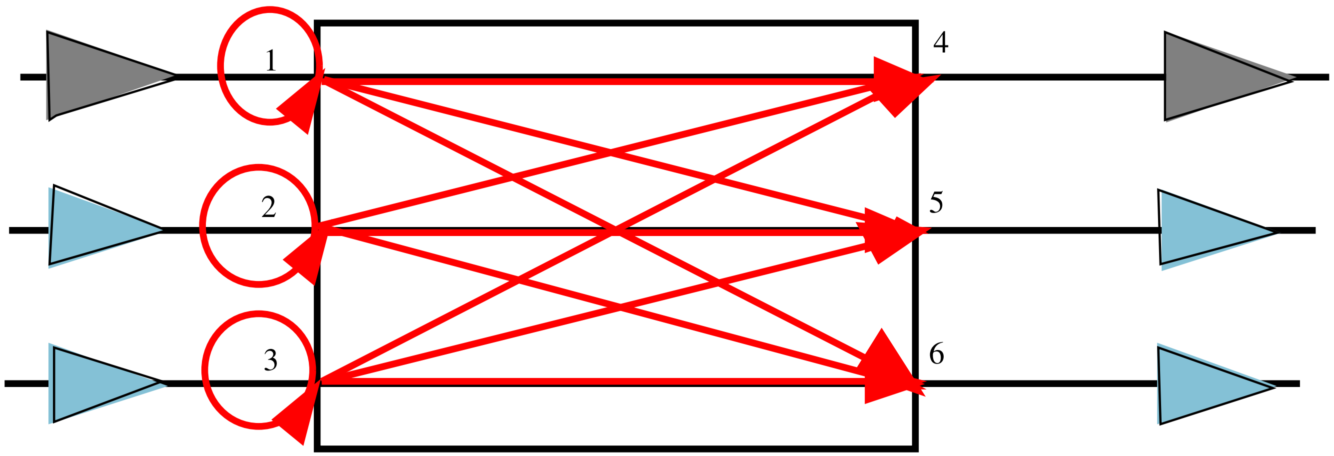
- Select clusters of potentially interfering nets
- Analyze large RC(LM) circuits plus drivers

Reduced-order modeling



- R.O.M. should behave like original circuit at I/O ports
- It should be used as a replacement for any analysis, such as delay, cross-talk, circuit-level (Spice), ...
- It should be much smaller than original circuit

Matrix-valued transfer function



- Linear 6-port is characterized by a 6×6-matrix-valued function:

$$\mathbf{Z}(s) = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) & \cdots & Z_{16}(s) \\ Z_{21}(s) & Z_{22}(s) & \cdots & Z_{26}(s) \\ \vdots & \vdots & \ddots & \vdots \\ Z_{61}(s) & Z_{62}(s) & \cdots & Z_{66}(s) \end{bmatrix}$$

Matrix versus scalar approximation

- Linear p-port is characterized by a $p \times p$-matrix-valued function:

$$\mathbf{Z}(s) = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) & \cdots & Z_{1p}(s) \\ Z_{21}(s) & Z_{22}(s) & \cdots & Z_{2p}(s) \\ \vdots & \vdots & \ddots & \vdots \\ Z_{p1}(s) & Z_{p2}(s) & \cdots & Z_{pp}(s) \end{bmatrix}$$

- Scalar approach: each of the p^2 entries $Z_{jk}(s)$ is approximated separately
- Matrix approach: one $p \times p$ matrix approximation for $\mathbf{Z}(s)$
- For reduced-order models based on Padé approximations:
  - Matrix approach is at least p times more efficient
  - For comparable accuracy, reduced-order models from matrix approach are smaller

Linear multi-port transfer function

- Circuit equations:

$$\mathbf{G}\,\mathbf{v} + \mathbf{C}\,\frac{d}{dt}\mathbf{v} = \mathbf{B}\,\mathbf{i}_{io} \qquad \xrightarrow{} \qquad \mathbf{G}\,\mathbf{V} + s\,\mathbf{C}\,\mathbf{V} = \mathbf{B}\,\mathbf{I}_{io}$$

$$\mathbf{v}_{io} = \mathbf{B}^{\top}\,\mathbf{v} \qquad \text{Laplace} \qquad \mathbf{V}_{io} = \mathbf{B}^{\top}\,\mathbf{V}$$

- Transfer function:

$$\mathbf{V}_{io} = \mathbf{Z}(s)\cdot\mathbf{I}_{io} \qquad \text{where} \qquad \mathbf{Z}(s) = \mathbf{B}^{\top}\,(\mathbf{G} + s\,\mathbf{C})^{-1}\,\mathbf{B}$$

- For multi-terminal RLC circuits:
  $\mathbf{G} = \mathbf{G}^{\top}$ and $\mathbf{C} = \mathbf{C}^{\top}$ are symmetric matrices

- For RC, RL, and LC circuits:
  $\mathbf{G}$ and $\mathbf{C}$ are also positive semi-definite

Need for reduced-order modeling

- Matrices $\mathbf{G}$ and $\mathbf{C}$ are $N \times N$ where N is large ($N \approx 10^{4-6}$)

$$\mathbf{Z}(s) = \boxed{\mathbf{B}^\top} \left(\boxed{\mathbf{G}} + s \boxed{\mathbf{C}} \right)^{-1} \boxed{\mathbf{B}}$$

- Reduced-order model with matrices of order $n \ll N$ ($n \approx 10^{0-2}$)

$$\mathbf{Z}_n(s) = \boxed{\mathbf{B}_n^\top} \left(\boxed{\mathbf{G}_n} + s \boxed{\mathbf{C}_n} \right)^{-1} \boxed{\mathbf{B}_n}$$

Reduced-order model in time domain

- Reduced-order impedance:

$$\mathbf{V}_{io}(s) = \mathbf{Z}_n(s)\,\mathbf{I}_{io}(s) = \mathbf{B}_n^\top \underbrace{(\mathbf{G}_n + s\,\mathbf{C}_n)^{-1}\,\mathbf{B}_n\,\mathbf{I}_{io}(s)}_{=\mathbf{X}(s)}$$

- Rewrite as follows:

$$\begin{aligned}(\mathbf{G}_n + s\,\mathbf{C}_n)\,\mathbf{X}(s) &= \mathbf{B}_n\,\mathbf{I}_{io}(s)\\ \mathbf{V}_{io}(s) &= \mathbf{B}_n^\top\,\mathbf{X}(s)\end{aligned}$$

- In time domain:

$$\begin{aligned}\mathbf{G}_n\,\mathbf{x}(t) + \mathbf{C}_n\,\frac{d}{dt}\mathbf{x}(t) &= \;\mathbf{B}_n\,\mathbf{i}_{io}(t)\\ \mathbf{v}_{io}(t) &= \mathbf{B}_n^\top\,\mathbf{x}(t)\end{aligned}$$

- Can be combined with the nonlinear equations of the circuit

Matrix-Padé approximation

- Given: $p \times p$-matrix-valued transfer function

$$\mathbf{Z}(s) = \mathbf{B}^{\mathsf{T}} (\mathbf{G} + s\,\mathbf{C})^{-1} \mathbf{B}$$

  where $\mathbf{G}$ and $\mathbf{C}$ are $N \times N$ matrices, and $\mathbf{B}$ is an $N \times p$ matrix

- More general case: $m \times p$-matrix-valued transfer function

$$\mathbf{Z}(s) = \mathbf{L}^{\mathsf{T}} (\mathbf{G} + s\,\mathbf{C})^{-1} \mathbf{B}$$

  where $\mathbf{B}$ is $N \times p$ and $\mathbf{L}$ is $N \times m$, for p-input m-output systems

- Choose expansion point $s_0 \in \mathbb{C}$ and expand about it:

$$\mathbf{Z}(s) = \mathbf{M}_0 + \mathbf{M}_1 (s - s_0) + \mathbf{M}_2 (s - s_0)^2 + \cdots + \mathbf{M}_k (s - s_0)^k + \cdots$$

- Coefficients $\mathbf{M}_k$'s are $p \times p$ matrices, the so-called *moments*

Matrix-Padé approximation, continued

- $p \times p$-matrix-valued transfer function of reduced-order model:

$$\mathbf{Z}_n(s) = \mathbf{B}_n^\top \left(\mathbf{G}_n + s\,\mathbf{C}_n\right)^{-1} \mathbf{B}_n$$

  where $\mathbf{G}_n$ and $\mathbf{C}_n$ are $n \times n$ matrices, and $\mathbf{B}_n$ is an $n \times p$ matrix

- $\mathbf{Z}_n$ is an *n-th matrix-Padé approximant* to $\mathbf{Z}$ if its expansion about s_0 matches as many leading moments of $\mathbf{Z}$ as possible:

$$\begin{aligned}\mathbf{Z}_n(s) &= \mathbf{M}_0 + \mathbf{M}_1(s-s_0) + \cdots + \mathbf{M}_{q-1}(s-s_0)^{q-1} + \widetilde{\mathbf{M}}_q(s-s_0)^q + \cdots \\ &= \mathbf{Z}(s) + \mathcal{O}\left(s^q\right)\end{aligned}$$

  where $q = q(n)$ is as large as possible

- $q(n) \geq 2\left\lfloor \dfrac{n}{p} \right\rfloor$ with equality if no *deflation* occurs

Computation of Padé approximants

- Standard textbook approach:
  - Explicitly compute first $q = q(n)$ moments

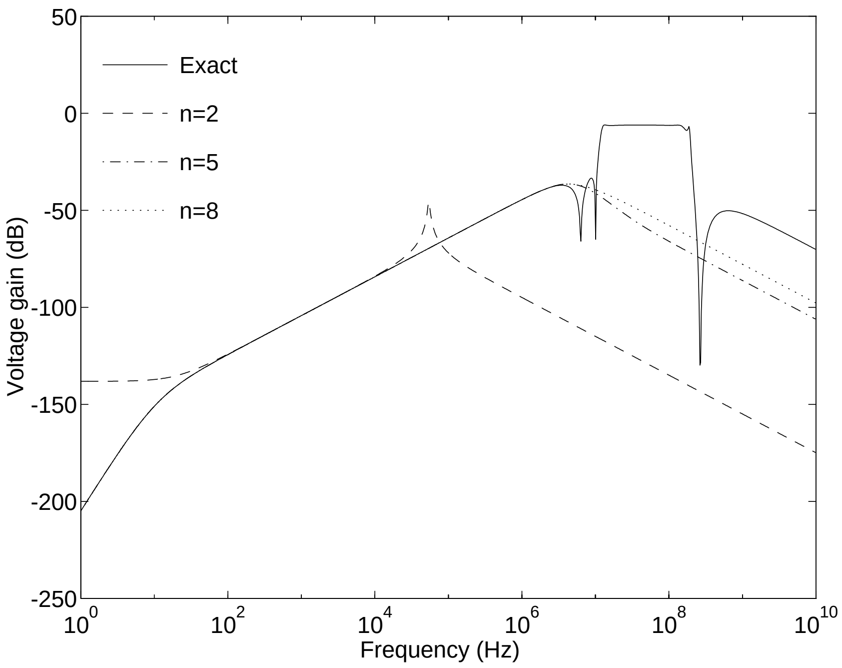
$$\mathbf{M}_0, \mathbf{M}_1, \ldots, \mathbf{M}_{q-1}$$

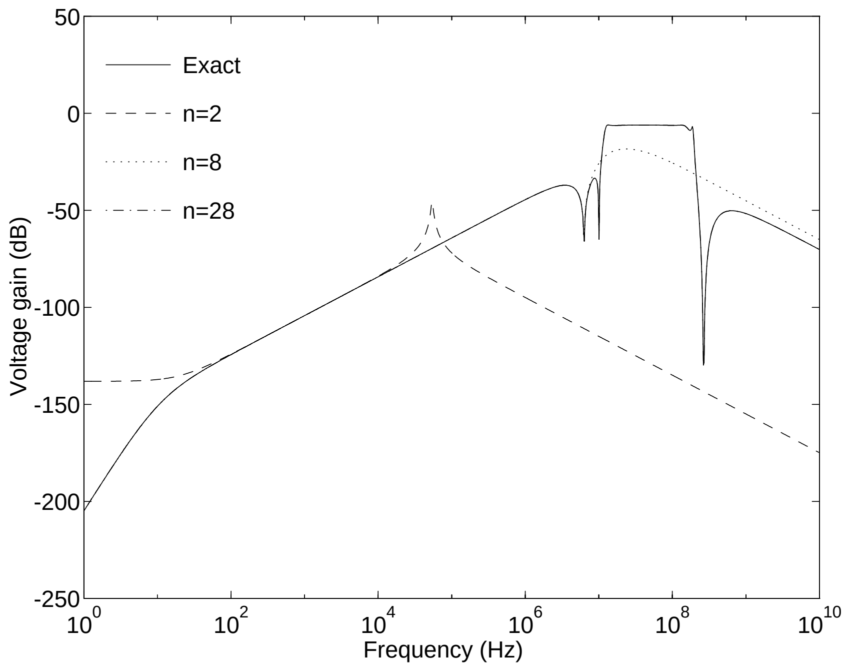
  - From these moments, obtain n-th Padé approximant $\mathbf{Z}_n$
- This is the approach used in AWE (for the case $p = 1$)
- Standard approach is inherently *numerically unstable*, even for moderately n
- An algorithm being numerically unstable means that, when run in finite-precision arithmetic, the accumulation of numerical round-off can (and usually will) lead to wrong results

Stable computation of Padé approximants

- Key is to avoid the explicit computation of the moments
- Instead, get the information contained in the first q moment form *modified moments* that are generated via suitable basis vectors from a Krylov-subspace method
- For Padé approximants, the *Lanczos process* (or a suitable extension thereof) is the natural Krylov-subspace method
- Computational costs are the same as for the (unstable) approaches based on explicit moment computation
- Lanczos-based reduced-order modeling techniques:
  - PVL for the case $p = 1$, SyPVL for linear RLC 1-ports
  - MPVL for the general case $p, m \geq 1$
  - SyMPVL for linear RLC p-ports, $p \geq 1$

Transfer function of a voltage filter $(p = 1)$





Explicit moments

Padé via Lanczos (PVL)

Computation via Lanczos

- Transfer function of RLC p-port:

$$\mathbf{Z}(s) = \mathbf{B}^\top (\mathbf{G} + s\,\mathbf{C})^{-1}\,\mathbf{B}$$

  where $\mathbf{G}$ and $\mathbf{C}$ are symmetric

- For simplicity, assume expansion point $s_0 = 0$

- Compute factorization

$$\mathbf{G} = \mathbf{U}^\top \mathbf{J}\,\mathbf{U}$$

  where $\mathbf{J}$ is “simple” (for RC, RL, LC circuits: $\mathbf{J} = \mathbf{I}$)

- With this factorization:

$$\begin{aligned}\mathbf{Z}(s) &= \mathbf{B}^\top \left(\mathbf{U}^\top \mathbf{J}\,\mathbf{U} + s\,\mathbf{C}\right)^{-1}\mathbf{B}\\ &= \mathbf{R}^\top (\mathbf{J} + s\,\mathbf{A})^{-1}\,\mathbf{R}\end{aligned}$$

$$\text{where} \quad \mathbf{A} = \mathbf{U}^{-\top}\,\mathbf{C}\,\mathbf{U}^{-1} \quad \text{and} \quad \mathbf{R} = \mathbf{U}^{-\top}\,\mathbf{B}$$

Exploiting symmetry

- Transfer function of general p-input m-output systems, after rewriting:

$$\mathbf{Z}(s) = \mathbf{L}^{\mathsf{T}} \left(\mathbf{J} + s\,\mathbf{A}\right)^{-1} \mathbf{B}$$

  where, in general, $\mathbf{A} \neq \mathbf{A}^{\mathsf{T}}$, and, usually, $\mathbf{J} = \mathbf{I}$

- Dominating computational cost of Lanczos-type method:

  Two matrix-vector products $\mathbf{A} \cdot \mathbf{v}$ and $\mathbf{A}^{\mathsf{T}} \cdot \mathbf{w}$ per step

- Transfer function of linear RLC p-port, after rewriting:

$$\mathbf{Z}(s) = \mathbf{R}^{\mathsf{T}} \left(\mathbf{J} + s\,\mathbf{A}\right)^{-1} \mathbf{R}$$

  where $\mathbf{A} = \mathbf{A}^{\mathsf{T}}$, and $\mathbf{J}$ is "simple"

- Dominating computational cost of Lanczos-type method:

  One matrix-vector product $\mathbf{A} \cdot \mathbf{v}$ per step

Computation via J-symmetric Lanczos

- Transfer function of linear RLC p-port:

$$\mathbf{Z}(s) = \mathbf{R}^\top (\mathbf{J} + s\,\mathbf{A})^{-1} \mathbf{R}$$

- Apply $\mathbf{J}$-symmetric Lanczos-type method to the symmetric matrix $\mathbf{A}$ and the block $\mathbf{R}$ of p starting vectors

- After n steps, the algorithm has generated n Lanczos vectors

$$\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$$

  that

  - are $\mathbf{J}$-orthogonal: $\mathbf{v}_j^\top \mathbf{J}\, \mathbf{v}_k = 0$ for all $j \neq k$

  - build a basis for the space spanned by the n first linearly columns of the block Krylov matrix

$$\left[\mathbf{J}^{-1}\mathbf{R} \;\; \left(\mathbf{J}^{-1}\mathbf{A}\right) \mathbf{J}^{-1}\mathbf{R} \;\; \cdots \;\; \left(\mathbf{J}^{-1}\mathbf{A}\right)^i \mathbf{J}^{-1}\mathbf{R} \;\; \cdots \right]$$

Lanczos in matrix form

- Matrix of first n Lanczos vectors:

$$\mathbf{V}_n = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \end{bmatrix}$$

- $\mathbf{J}$-orthogonality:

$$\mathbf{V}_n^{\mathsf{T}} \, \mathbf{J} \, \mathbf{V}_n = \boxed{\mathbf{\Delta}_n} = \text{diagonal matrix}$$

- Corresponding projection of $\mathbf{A}$ onto Lanczos vectors:

$$\boxed{\mathbf{T}_n} = \boxed{\mathbf{\Delta}_n^{-1}} \, \boxed{\mathbf{V}_n^{\mathsf{T}}} \, \boxed{\mathbf{A}} \, \boxed{\mathbf{V}_n}$$

Lanczos recurrences in matrix form

- At step n, we generate the n-th Lanczos vector $\mathbf{v}_n$
- After $n-1$ steps:

$$\mathbf{A}\,\mathbf{V}_{n-1} = \mathbf{V}_{n-1}\,\mathbf{T}_{n-1} + \Big[\underbrace{\mathbf{0} \ \cdots \ \mathbf{0}}_{n-1-p_c} \ \ \underbrace{\hat{\mathbf{v}}_n \ \cdots \ \hat{\mathbf{v}}_{n-1+p_c}}_{p_c}\Big] + \mathbf{V}_{\mathsf{defl}}$$

- *Deflation*:
  If $\hat{\mathbf{v}}_n \approx \mathbf{0}$, set $\mathbf{V}_{\mathsf{defl}} = \big[\mathbf{V}_{\mathsf{defl}} \ \ \hat{\mathbf{v}}_n\big]$ and $p_c = p_c - 1$
  (Initially: $p_c = p$)
- Otherwise, $\hat{\mathbf{v}}_n$ becomes the n-th Lanczos vector $\mathbf{v}_n$.
  Set $\mathbf{V}_n = \big[\mathbf{V}_{n-1} \ \ \mathbf{v}_n\big]$ and update the recurrence relation:

$$\mathbf{A}\,\mathbf{V}_n = \mathbf{V}_n\,\mathbf{T}_n + \Big[\underbrace{\mathbf{0} \ \cdots \ \mathbf{0}}_{n-p_c} \ \ \underbrace{\hat{\mathbf{v}}_{n+1} \ \cdots \ \hat{\mathbf{v}}_{n+p_c}}_{p_c}\Big] + \mathbf{V}_{\mathsf{defl}}$$

Lanczos-Padé connection

- During first p Lanczos steps, we $\mathbf{J}$-orthogonalize the starting vectors:

$$\mathbf{J}^{-1}\mathbf{R} = \mathbf{V}_p\,\boldsymbol{\rho}$$

- In terms of matrices $\mathbf{T}_n$, $\boldsymbol{\Delta}_n$, $\boldsymbol{\rho}$ from n Lanczos steps, the n-th Padé approximant $\mathbf{Z}_n$ to $\mathbf{Z}$ is given by

$$\mathbf{Z}_n(s) = \mathbf{B}_n^{\mathsf{T}}\left(\boldsymbol{\Delta}_n^{-1} + s\,\mathbf{T}_n\,\boldsymbol{\Delta}_n^{-1}\right)^{-1}\mathbf{B}_n, \quad \mathbf{B}_n = \begin{bmatrix} \boldsymbol{\rho} \\ \mathbf{0}_{n-p\times p} \end{bmatrix}$$

- For RC, RL, and LC circuits:

$$\mathbf{Z}_n(s) = \mathbf{B}_n^{\mathsf{T}}\left(\mathbf{I} + s\,\mathbf{T}_n\right)^{-1}\mathbf{B}_n$$

  and $\mathbf{T}_n$ is symmetric positive definite

Summary of SyMPVL for RLC p-ports

- Given: Transfer function $\mathbf{Z}(s) = \mathbf{B}^\top (\mathbf{G} + s\,\mathbf{C})^{-1}\,\mathbf{B}$
- Compute factorization

$$\mathbf{G} = \mathbf{U}^\top \mathbf{J}\,\mathbf{U}$$

- Apply $\mathbf{J}$-symmetric Lanczos method to

$$\mathbf{A} = \mathbf{U}^{-\top}\,\mathbf{C}\,\mathbf{U}^{-1} \quad \text{and} \quad \mathbf{R} = \mathbf{U}^{-\top}\,\mathbf{B}$$

- After n $(\geq p)$ steps, we have $\mathbf{T}_n$, $\mathbf{\Delta}_n$, $\mathbf{B}_n$ and these define n-th Padé approximant

$$\mathbf{Z}_n(s) = \mathbf{B}_n^\top \left(\mathbf{\Delta}_n^{-1} + s\,\mathbf{T}_n\,\mathbf{\Delta}_n^{-1}\right)^{-1} \mathbf{B}_n$$

- Check error of reduced-order model (e.g., use bounds due to Bai and Ye, '97)
- If necessary, continue Lanczos method

Stability and passivity

- Original RLC port is stable and passive
- A non-passive reduced-order model may cause circuit to be unstable
- Therefore, reduced-order model should preserve stability and passivity

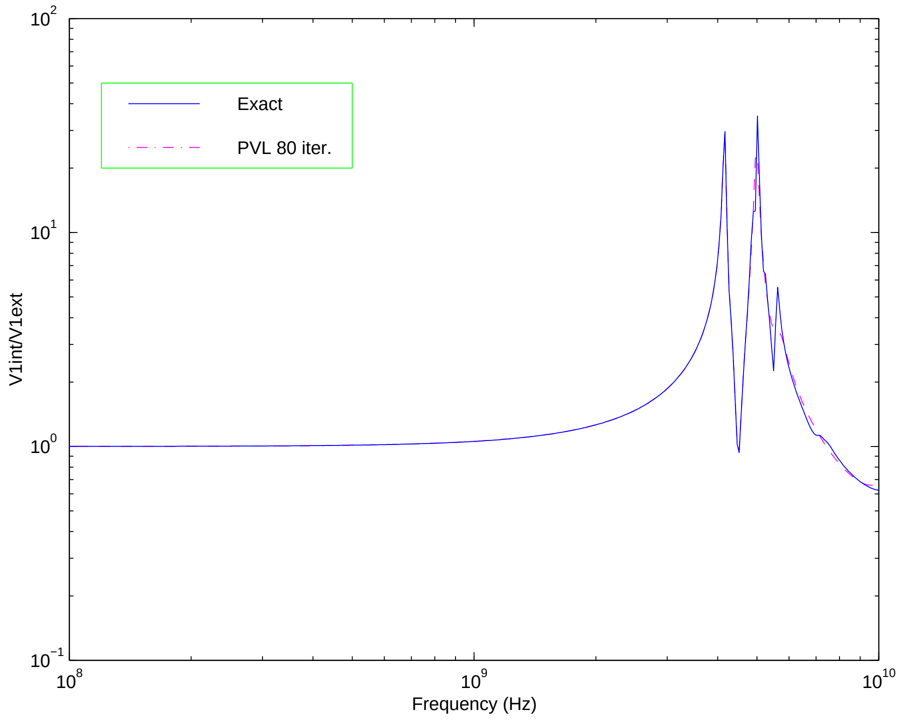
True Padé approximations

- Methods based on Padé approximations, such as SyMPVL:
  - In theory, reduced-order models are stable and passive for RC, RL, and LC circuits
  - Careful implementation is needed to ensure these properties in practice!
  - Preservation of stability and passivity is not guaranteed for RLC circuits. In general, a suitable post-processing is necessary
- n-th reduced-order model is unique and does not depend on the chosen formulation of the circuit equations

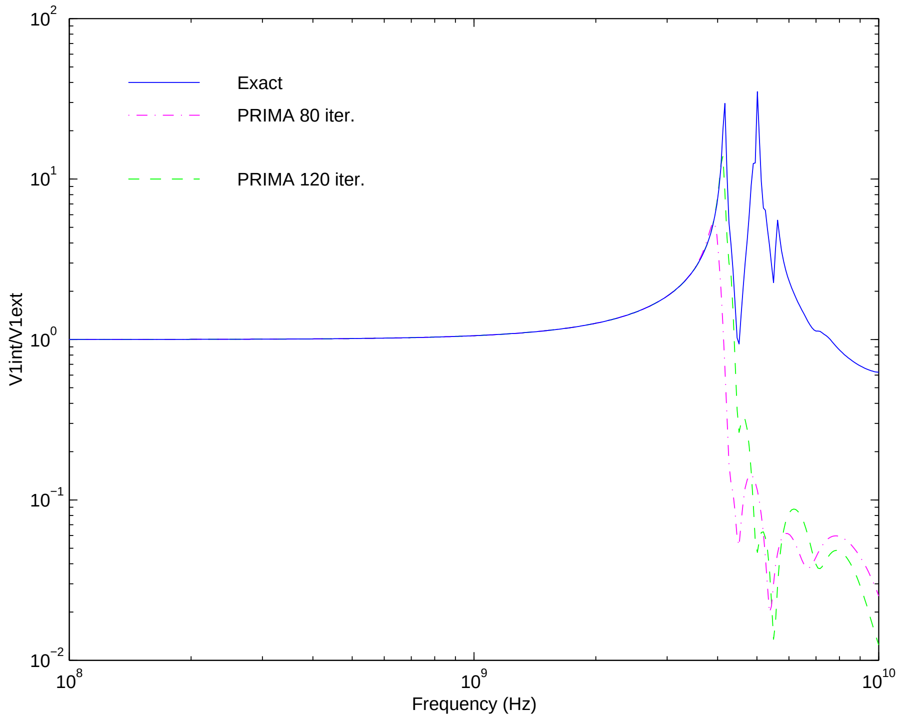
Padé-type approximations

- Padé-type approaches, such as PRIMA: trade accuracy for—in theory—stable and passive reduced-order models
  - In theory, reduced-order models are stable and passive for RLC circuits
  - However, one trades in half of the possible accuracy. More precisely, reduced-order models of order n match only $\left\lfloor \frac{n}{p} \right\rfloor$ moments instead of $2 \left\lfloor \frac{n}{p} \right\rfloor$ moments for true Padé-based reduced-order models of order n
  - There are unresolved implementation issues
- Reduced-order model strongly depends on the chosen formulation of the circuit equations

True Padé versus Padé-type approximations



SyMPVL

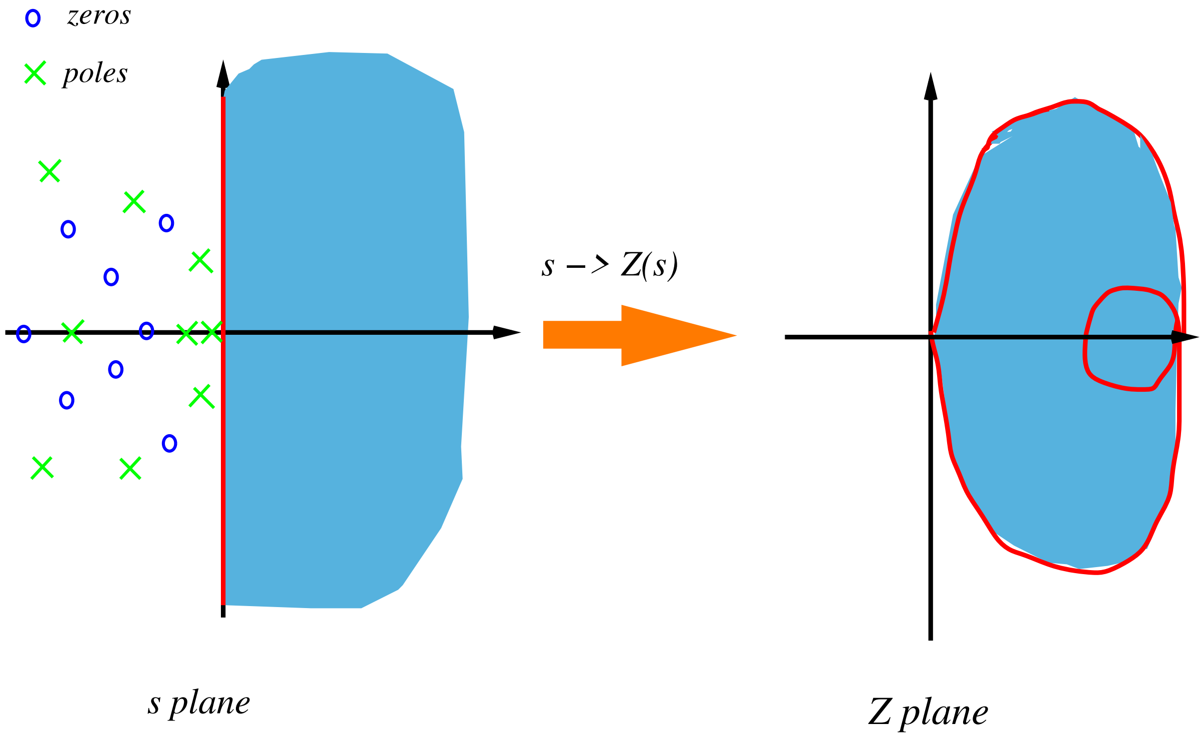


PRIMA

Stability and passivity, definitions

- A circuit is *passive* if none of its elements generates energy
- A linear RLC p-port is passive if its transfer function $\mathbf{Z}$ is *positive real*:
  - $\mathbf{Z}$ has no poles in $\mathbb{C}_+ = \{\, s \in \mathbb{C} \mid \texttt{Real part of } s > 0 \,\}$
  - $\mathbf{Z}$ is real for real s
  - $\texttt{Real part of } \mathbf{z}^{\mathsf{H}}\,\mathbf{Z}(s)\,\mathbf{z} \geq 0$ for all $s \in \mathbb{C}_+$ and $\mathbf{z} \in \mathbb{C}^p$
- A linear RLC p-port is *stable* if its transfer function $\mathbf{Z}$ has no poles in $\mathbb{C}_+$ and if possible poles on the imaginary axis are simple
- Passivity implies stability

Passivity (for $p = 1$)



Real part of $\mathbf{Z}(s) \geq 0$ for all $s \in \mathbb{C}$ with Real part of $s \geq 0$

Stability and passivity, continued

- For RC, RL, and LC circuits:

$$\mathbf{Z}_n(s) = \mathbf{B}_n^{\mathsf{T}} \left(\mathbf{I} + s\, \mathbf{T}_n \right)^{-1} \mathbf{B}_n$$

  and $\mathbf{T}_n$ is symmetric positive definite

- Using the positive definiteness of $\mathbf{T}_n$, it is straightforward to verify that $\mathbf{Z}_n$ is positive real and thus stable and passive

- The Lanczos method needs to be implemented carefully so that $\mathbf{T}_n$ is indeed positive definite in finite-precision arithmetic (Bai, Feldmann, and Freund, ‘98)

- The key is to implement the Lanczos method with coupled recurrences

Coupled recurrences

- In addition to $\mathbf{V}_n$, second set of basis vectors

$$\mathbf{P}_n = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_n \end{bmatrix}$$

- Recurrences in matrix form:

$$\mathbf{A}\,\mathbf{P}_n = \mathbf{V}_n\,\mathbf{L}_n + \Big[\underbrace{\mathbf{0} \;\; \cdots \;\; \mathbf{0}}_{n-p_c} \;\; \underbrace{\hat{\mathbf{v}}_{n+1} \;\; \cdots \;\; \hat{\mathbf{v}}_{n+p_c}}_{p_c}\Big] + \mathbf{V}_{\mathsf{defl}}$$

$$\mathbf{V}_n = \mathbf{P}_n\,\mathbf{L}_n^{\top}$$

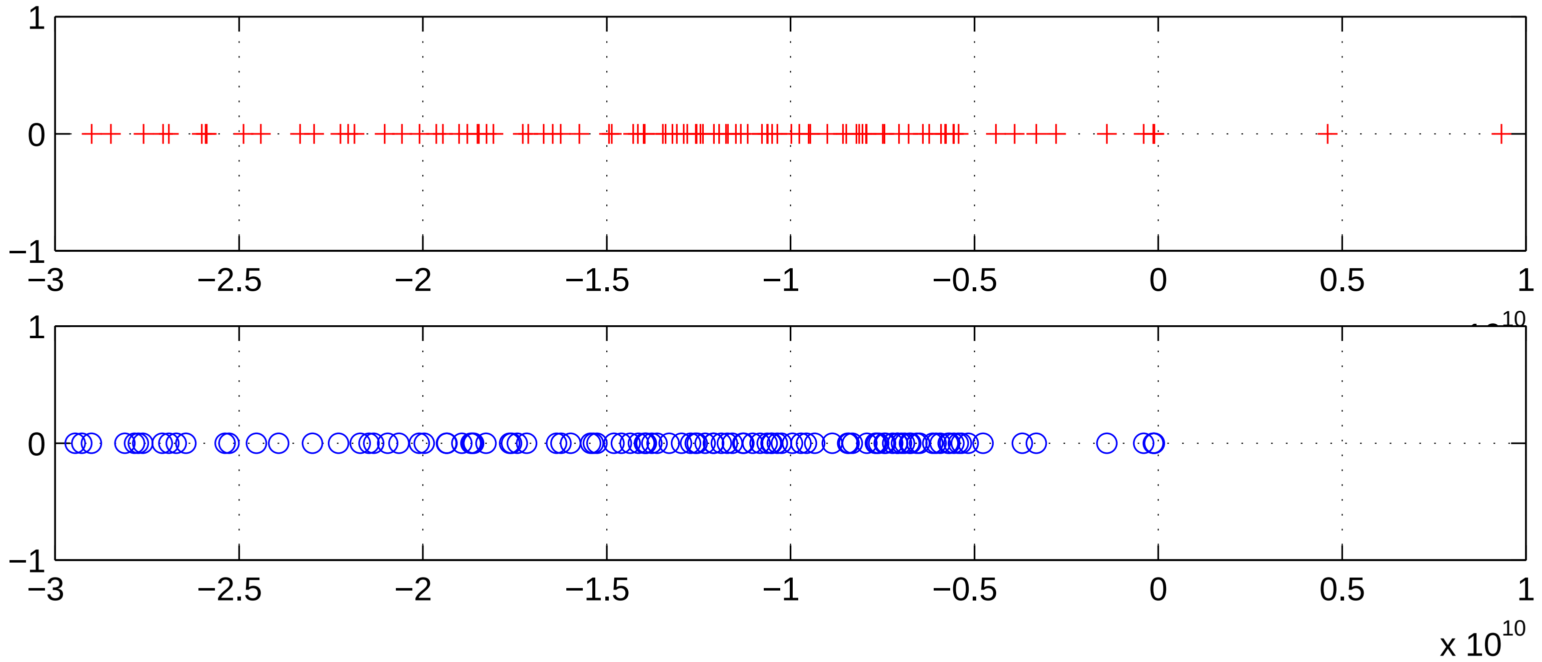
- $\mathbf{T}_n$ is then computed via

$$\mathbf{T}_n = \mathbf{L}_n\,\mathbf{L}_n^{\top}$$

  and this guarantees positive definiteness of $\mathbf{T}_n$

Simulation of large RC circuit

- $\approx 200,000$ RC elements, $p = 150$ I/O ports
- Reduced-order model of order $n = 300$



Poles from non-coupled (+) and coupled (O) recurrences

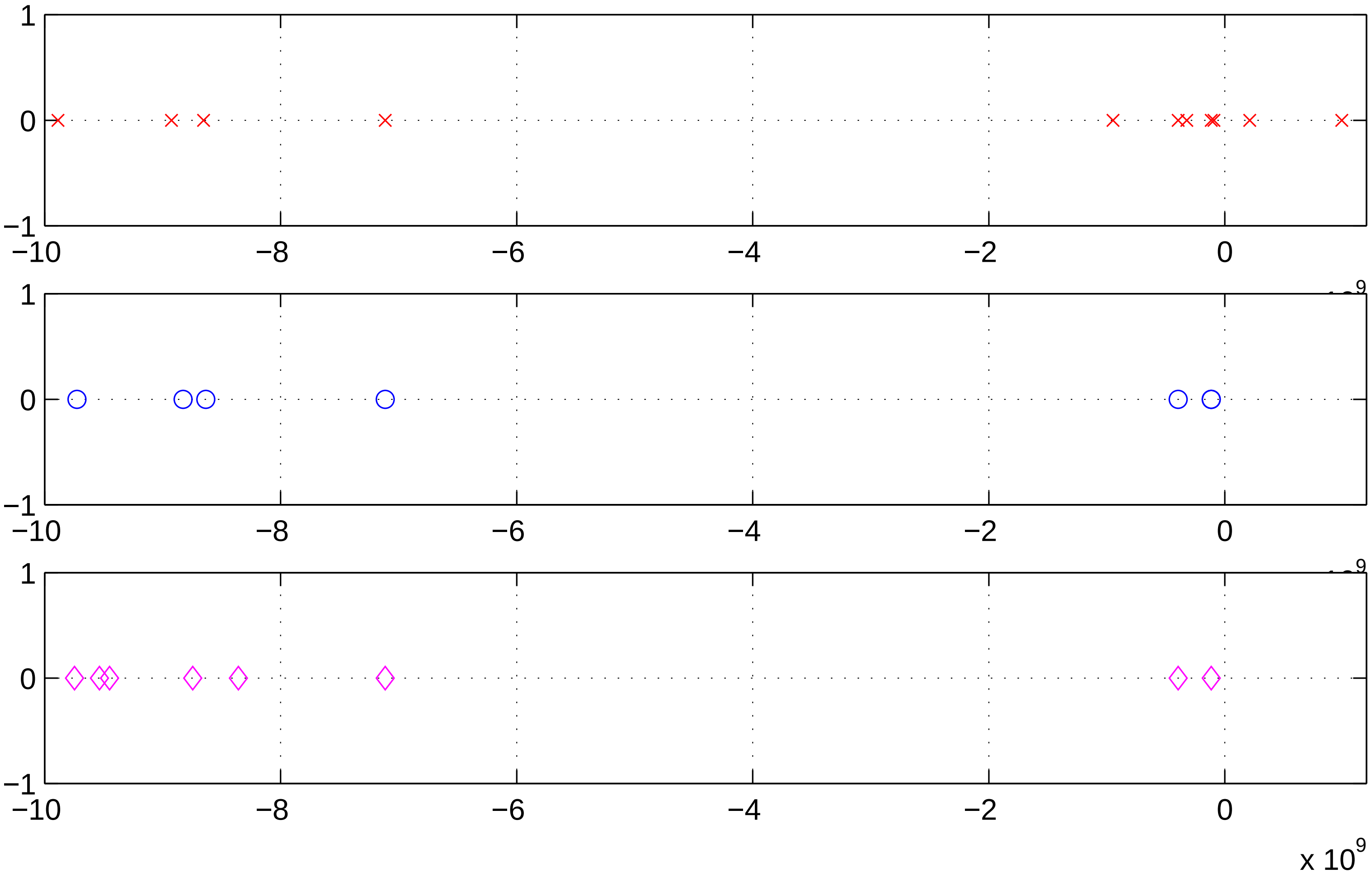
A second example

- $\approx 30,000$ RC elements, $p = 30$ I/O ports
- Reduced-order model of order $n = 60$

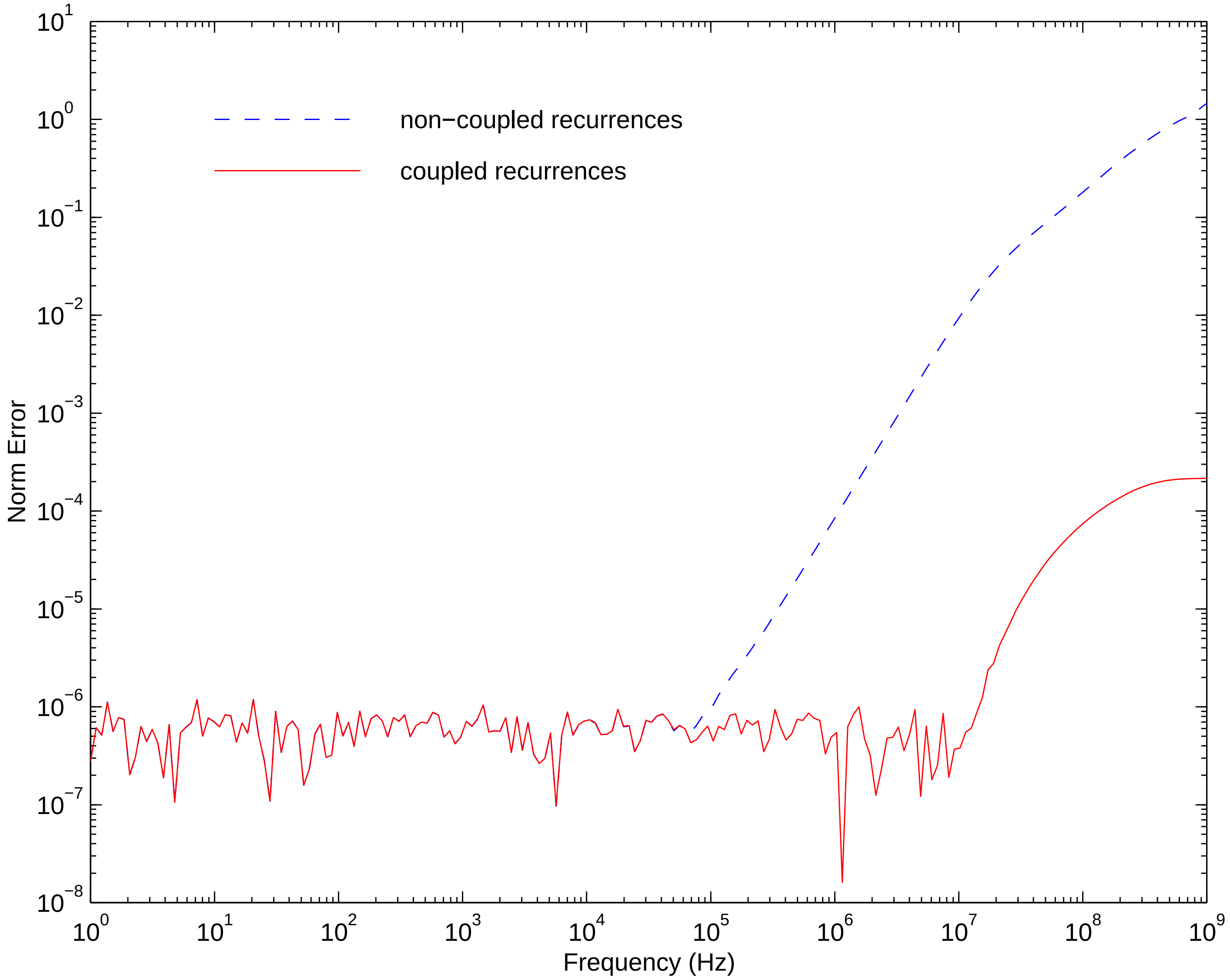
Poles from non-coupled recurrences

Poles from coupled recurrences

Exact poles



Second example: frequency-domain error



Provable passive reduced-order models

- Transfer function of RLC p-port:

$$\mathbf{Z}(s) = \mathbf{B}^\top (\mathbf{G} + s\,\mathbf{C})^{-1}\,\mathbf{B}, \quad \text{where} \quad \mathbf{G} = \mathbf{G}^\top \quad \text{and} \quad \mathbf{C} = \mathbf{C}^\top$$

- For modified nodal analysis, equivalent representation:

$$\mathbf{Z}(s) = \tilde{\mathbf{B}}^\top \left(\widetilde{\mathbf{G}} + s\,\tilde{\mathbf{C}}\right)^{-1} \tilde{\mathbf{B}}$$

where

$$\widetilde{\mathbf{G}} = \mathbf{Q} + \mathbf{N}, \quad \mathbf{Q} = \mathbf{Q}^\top, \quad \mathbf{N} = -\mathbf{N}^\top, \quad \tilde{\mathbf{C}} = \tilde{\mathbf{C}}^\top$$

and $\mathbf{Q}$ and $\tilde{\mathbf{C}}$ are positive semi-definite

- Choose *any* $N \times n$ matrix $\mathbf{V}_n$ and project:

$$\mathbf{Z}_n^{(\text{pas})}(s) = \mathbf{B}_n^\top (\mathbf{G}_n + s\,\mathbf{C}_n)^{-1}\,\mathbf{B}_n$$

$$\text{where} \quad \mathbf{G}_n = \mathbf{V}_n^\top\,\widetilde{\mathbf{G}}\,\mathbf{V}_n, \quad \mathbf{C}_n = \mathbf{V}_n^\top\,\tilde{\mathbf{C}}\,\mathbf{V}_n, \quad \mathbf{B}_n = \mathbf{V}_n^\top\,\tilde{\mathbf{B}}$$

Properties of $\mathbf{Z}_n^{(\mathrm{pas})}$

- Need to assume that the matrix $\mathbf{G}_n$ is nonsingular, and thus there is a breakdown condition!
- Using the matrix properties of $\widetilde{\mathbf{G}} = \mathbf{Q} + \mathbf{N}$ and $\tilde{\mathbf{C}}$, it is straightforward to prove that $\mathbf{Z}_n^{(\mathrm{pas})}$ is passive
- Choice of the matrix $\mathbf{V}_n$?
- PRIMA (Odabasioglu,'96, Odabasioglu, Celik, and Pileggi, '97):
  - Obtain $\mathbf{V}_n$ by running n steps of block Arnoldi applied to the matrix $\left(\widetilde{\mathbf{G}}\right)^{-1} \tilde{\mathbf{C}}$ and the starting block $\left(\widetilde{\mathbf{G}}\right)^{-1} \tilde{\mathbf{B}}$
  - The reduced-order model then matches the first $\left\lfloor \frac{n}{p} \right\rfloor$ moments (and thus half as many as SyMPVL)

Only the space spanned by $\mathbf{V}_n$ matters

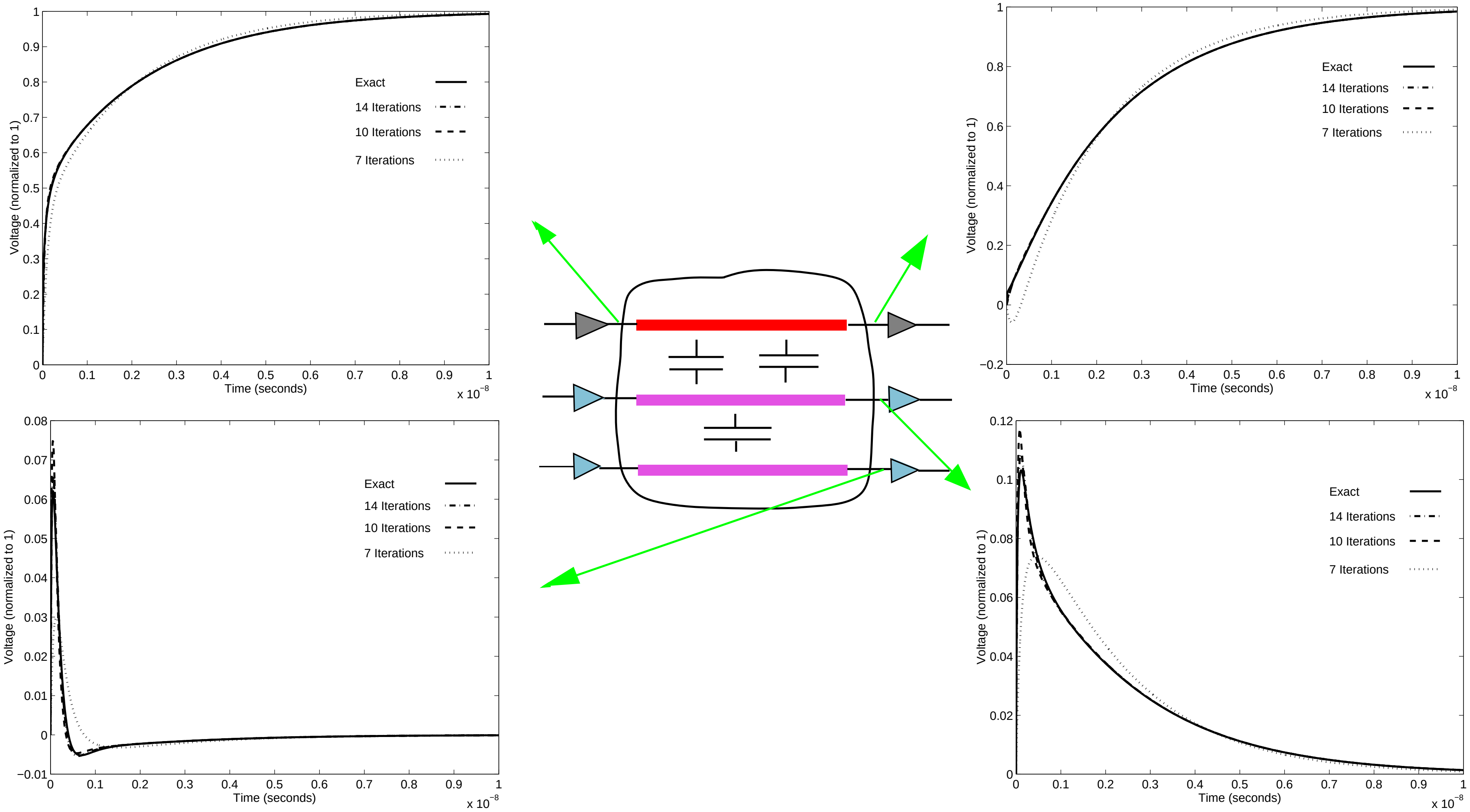
- The provable passivity of $\mathbf{Z}_n^{(\text{pas})}$ does not come from the block Arnoldi method, but is due to the space spanned by the columns of $\mathbf{V}_n$
  (Freund, '98)

- This space is just the n-th block Krylov subspace, which also underlies SyMPVL

- Using the projection idea, one can thus get a provably passive reduced-order model from SyMPVL, in addition to the Padé model generated by SyMPVL, at very little extra computational costs

- If one only cares about computing $\mathbf{Z}_n^{(\text{pas})}$, block Arnoldi is not the best method
  (Freund, '98)

Delay and cross-talk



Concluding remarks

- Reduced-order modeling is key to interconnect analysis
- SyMPVL is an efficient engine for computing reduced-order models of large RLC circuits
- Stability and passivity is guaranteed for RC, RL, and LC circuits
- There are variants that produce stable and passive models for general RLC circuits
- SyMPVL can be used for both time-domain and frequency-domain analysis
- PRIMA trades accuracy for theoretical passivity
- For RC, RL, and LC circuits, SyMPVL certainly is the method of choice