Fields and Poynting Vector in and near the focal plane of a slab with index of refraction close to -1 for a line source

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This paper presents numerical integral calculations of the exact 2D fields transmitted through a planar slab due to a magnetic current line source on the opposite side. This source gives a delta function in the electric field component in the plane of the source. We consider cases where relative permittivity ϵ_r and relative permeability μ_r are both close to -1, but further where the real part of the index of refraction n is very close or equal to -1. Results for a (real) electric current line source may be obtained by interchanging ϵ_r and μ_r. We show that focusing in the transverse direction is very different from the focusing in the propagation direction. The transverse width of the focal spot depends on ϵ_r, μ_r and slab thickness and is subwavelength in size for these cases. However, the width in the propagation direction is most strongly dependent on the slab thickness and is not subwavelength. Quiver plots of the Poynting vector illustrate an energy flow that generally looks like what would be expected for a real focus when slab thickness is greater than a few wavelengths. For thin slabs the transmitted intensity falls off monotonically with distance from the slab in the axis plane.

Introduction

Propagation and refraction in and through a negative-index medium (NIM) and indeed the existence of such a material are topics of major interest now mainly due to seminal work by Pendry et al. [1], Pendry [2] and Smith et al. [3]. They were aware that Veselago [4] first considered the possibility of a negative-index material and described its properties as being left handed because the normally right-handed triad of vectors of a plane wave (propagation vector $\vec{k}$, electric field $\vec{E}$, and magnetic field vector $\vec{H}$) becomes left handed in a NIM. Pendry's paper [2] provided an analytic demonstration that in the limit as both ϵ_r and μ_r approach -1, a planar slab of thickness d_2 provides transmission with a very special focusing property. The limit transmission coefficient provides a perfect focus of a source located at a distance $d_1 < d_2$ on one side into a plane located at distance $d_2 - d_1$ from the opposite face. (See Fig. 1(A).) Perfect focus means no spread of a point due to the usual finite range of wave numbers of a normal lens contributing to the image. This result created a rush to understand how this rather counterintuitive result could be. The interest in a NIM was further enhanced by the experiment [3], which demonstrated negative refraction from a metamaterial.

Papers that have followed in the general investigation of NIM have ranged from investigation of how to design metamaterials with a desired permittivity and permeability to calculation of properties of ideal NIM. We have been working on the kinds of waves created at the boundary of a half-space of NIM [5] and a description of the negative refraction of a wave packet (a beam wave) [6]. We present here detailed calculations to illustrate the approximate focusing of fields from a point (line) source with what might be realistic deviations of ϵ_r and μ_r from -1.

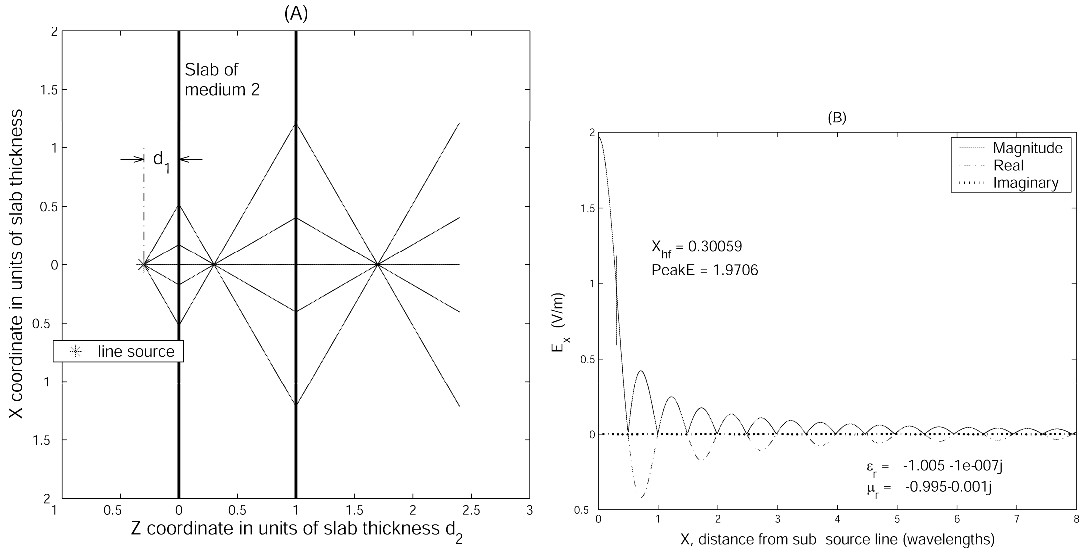

Figure 1: (A) Coordinate system for field calculations and illustration of geometric focusing for $n_2 = -1$. Line source is in the Y direction. Geometry shows that focal point to right of slab is at distance $d_2 - d_1$ from the slab. (B) The transmitted field component E_x in the image plane for a slab thickness of $d_2 = 5\lambda_o$, source distance = image distance = $2.5\lambda_o$. X_{hf} denotes the value of x where $|E_x|$ drops to 0.5 of its maximum.

Formulation of the analytic and numerical problem

We take a line source in the Y direction (normal to the plane of Fig. 1) located at $z = -d_1$, $x = 0$. The uniform isotropic slab (medium 2) with relative permittivity ϵ_{2r} and permeability μ_{2r} is located between the planes $z = 0$ and $z = d_2$. The magnetic line current is normalized for convenience to be $J_m = -2\delta(x)\delta(z + d_1)$ with time dependence $\exp(j\omega t)$. Then an exact solution of Maxwell's equations for incident and reflected fields in all the regions can be expressed simply by Fourier transform methods. This source creates a p-polarized wave with components E_x, E_z and H_y. The transmitted E_x in region 3 ($z > d_2$) is given by

$$E_x^{tr} = \frac{1}{2\pi} \int_{-\infty}^{\infty} T(k_x) \exp\left(-jk_{z1}(z - d_2 + d_1) - jk_x x\right) dk_x \tag{1}$$

where $k_{z1} = \sqrt{k_0^2 - k_x^2}$, and $k_o = \omega/c$. $T(k_x)$ is the overall transmission coefficient of the slab for a plane wave with incidence described by the propagation vector components (k_x, k_{z1}). There is a symmetry between the cases of s and p polarization. We have chosen p polarization as mentioned above. The index of refraction of medium 2 is $n_2 = \sqrt{\epsilon_{2r}\mu_{2r}}$ and here the square root with negative imaginary part must be chosen in our convention. The equation for $T(k_x)$ applies in cases where $k_x > k_o$; i.e., for the evanescent wave components, and for these waves k_{z1} must be chosen to have a negative imaginary part. The other field components, all related by the frequency domain equation $j\omega\epsilon\vec{E} = \vec{k} \times \vec{H}$, are calculated from Fourier transforms similar to (1), but with additional factors of $(-k_x/k_{z1})$ or $-k_{z1}/(j\omega\epsilon)$ inside the integrand for E_z or H_y respectively.

Field Components

Most of our effort has been devoted to calculations of E_x because this component is the one that focuses to a delta function in the limit $\epsilon_{2r} \to -1$ and $\mu_{2r} \to -1$. As heuristic motivation, we note that when $\Re(n_2)$ deviates significantly from -1, the geometric focusing illustrated in Fig. 1(A) becomes chaotic in appearance. Detailed calculations based on (1) show that the focus degrades severely because the magnitude of E_x (and of the other components) oscillates strongly with x and the maximum peak often does not occur at x = 0. Hence, we have carried out many series of calculations where $\Re(\epsilon_{2r})$ and $\Re(\mu_{2r})$ follow a hyperbolic trajectory in the $\Re(\epsilon_{2r})$ – $\Re(\mu_{2r})$ plane determined by $\Re(n_2) = -1$ with fixed imaginary parts of ϵ_{2r} and μ_{2r}. We use the term image plane to mean the plane of the idealized limit focus for $n = -1$. All of our calculations were done for the case $d_1 = (1/2)d_2$. This case seems useful because both the object and image have good clearance from the slab.

As an example of a detailed calculation of E_x in the image plane, Fig. 1(B) shows the result for a case where the loss is mostly associated with μ_{2r}. Understanding of several features of this plot is explained by asymptotic analysis, which will be presented in detail in another paper. The most important property of the asymptotic spectrum of E_x (the integrand in (1) except for the factor $\exp(-jk_x x)$) is that it stays approximately flat out to the wavenumber k_p where $k_p^2 = k_o^2 + (M/d_2)^2$ with $M = \ln(2/|(\delta_e)|)$ and $\delta_e = \epsilon_{2r} + 1$ and then the spectrum decreases rapidly as $\exp(-k_x d_2)$ for k_x fairly large compared to k_p. Thus, the amplitude as a function of x in the image plane is roughly proportional to $\sin(k_p x)/x$. In simplest terms, the peak gets sharper in this transverse direction as M/d_2 gets large. M grows logarithmically (slowly) as $\epsilon_{2r} \to -1$. However, we find that for values of $d_2 \lesssim 1$ (measured in units of wavelength), the amplitude of E_x falls off monotonically with distance from the exit plane. In other words, for small d_2, there is no peak in the Z direction. Hence, Fig. 1(B) is chosen to show a case where $d_2 = 5$. The choice of $\delta_e = -0.005 - 10^{-7}j$ makes $k_p = 6.396$ (reciprocal wavelength units). Since the function $\sin(u)/u$ falls to 0.5 of its peak at $u = 1.896$, the asymptotic prediction for the half width of the peak is $1.896/6.396 = 0.296$, which is quite close to the value of 0.30059 obtained from the detailed integration. Thus, the total transverse width at half maximum is about $0.6\lambda_o$, which is clearly subwavelength.

For the same case Fig. 2(A) shows the magnitude of E_x along the Z axis (the direction of propagation perpendicular to the slab). Fig. 2(A) shows that the Z-direction focal width, as measured by the width at half height, is greater by a factor of about 3.3 than that in the X direction and is not subwavelength. We have found in all the cases we have run that the focal width in the Z direction depends mostly on the slab thickness and relatively little on the deviations of ϵ_{2r} and μ_{2r} from -1. Furthermore, the Z-direction focal width is within 1 percent the same for a slab of thickness $10\lambda_o$ as that shown for $5\lambda_o$. We also calculated E_z and H_y for this case. Fig. 2(B) shows the pattern of the resulting Poynting vector.

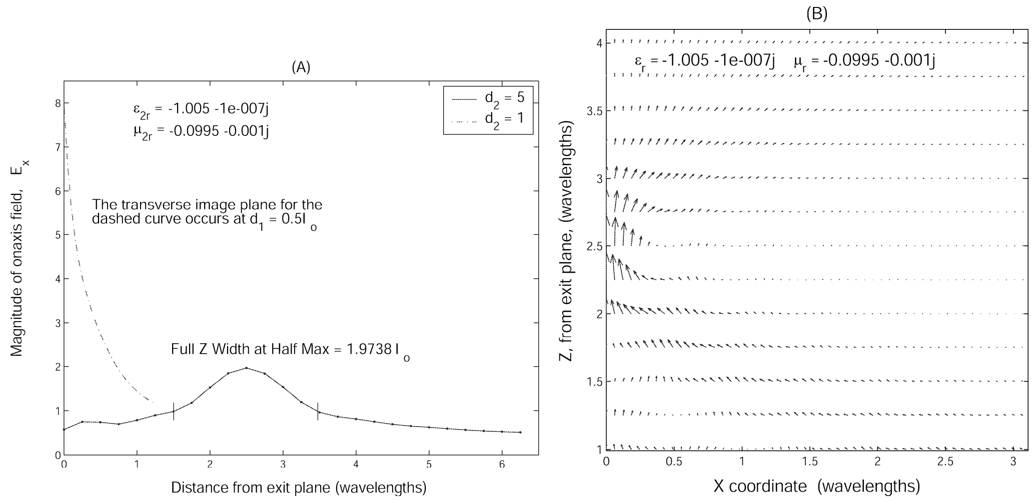

Figure 2: (A) The transmitted field component $|E_x|$ on the Z axis. The $d_2 = 1\lambda_o$ case decreases monotonically. (B) Quiver plot of the Poynting vector for $d_2 = 5$.

Conclusion

We have illustrated the limitation on focusing of the E_x component for a thick and thin slab. Similar calculations have generated all components for many cases. Quiver plots of the transmitted Poynting vector illustrate the focusing in another way and show again that for thin slabs there is no focusing in the Z direction. Plots of the X-direction focal width versus the deviation of ϵ_{2r} or μ_{2r} from -1 summarize many runs and show approximate agreement with the asymptotic estimate.

References

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